Triangle to Sine Conversion with OTAs

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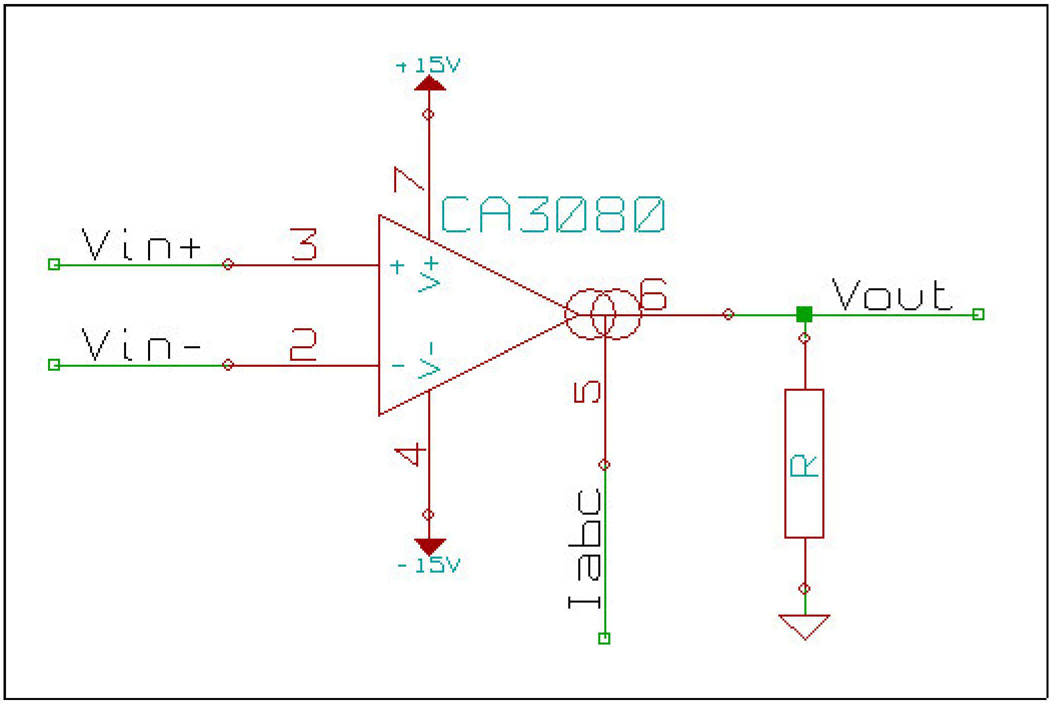
This note describes the basic operation of a triangle to sine converter using OTAs. I have made no attempts to show a complete design, but only to explain and verify the operation of such a circuit. I have not actually designed my own converter, but have built one (Oakley VCO). The diagram below shows possible pin connections to the CA3080. The Vin inputs will need attenuators and Vout will need a buffer.



definitions:

Vin = (Vin+) - (Vin-)

Iabc = amplifier bias current

u(t) is the unit step function. (0 for t<0, 1 for t>0)

$\mathrm{uA} := 10^{-3}\mathrm{mA}$

$u(t) := \Phi(t)$

$\mathrm{Kohm} := 10^{3}\mathrm{ohm}$

$I_{abc} := 500\mathrm{uA}$

$V_t := 0.026\mathrm{V}$

$\mathrm{dB} := 1$

First, I'll start with the basic OTA equation. A CA3080 is the right component, but since it is gone now, one half of an LM13700 would work fine. These are cheap, and the NJM13700 is an even cheaper replacement. I am not going into the derivation of this equation because it has been done many times. A good exersize is to do it on your own. It helped me when I was first learning about this type of circuit. look to my webpage for links to a very complete OTA analysis. For a triangle to sine converter, the linearized (small signal) equation will not work. The OTA must be overdriven, so the nonlinear voltage-current relationship should be used.

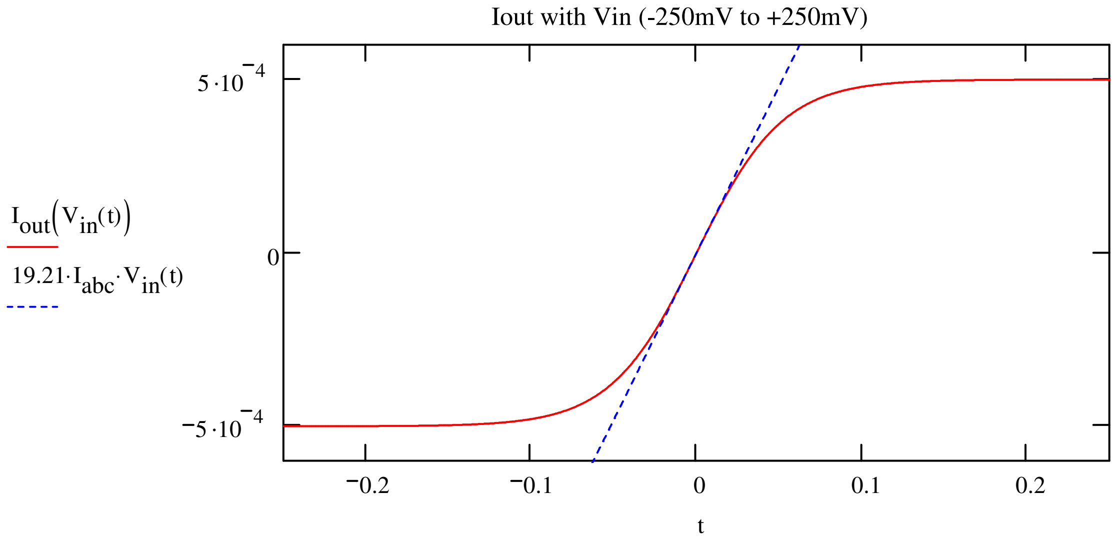
$$I_{out}(V_{in}) := I_{abc} \cdot \tanh\left(\frac{V_{in}}{2 \cdot V_t}\right)$$

The next plot shows the output current with an input voltage swept from -0.25V to +0.25V. This shows how the the amplifier distorts for large signal differences at Vin inputs. If you are not familair with the unit step function, then the definition below for Vin may look like nonsense, but it defines an input of:

-0.25V	for	t = -infinity to -0.25sec.
t Volts	for	t = -0.25sec to +0.25sec
+0.25V	for	t = +0.25sec to infinity.

As a reference, I have also shown the linear approximation of the output current.

$$V_{in}(t) := \left[t \cdot (u(t+0.25) - u(t-0.25)) - 0.25 \cdot \left(u\left(t+\infty\right) - u(t+0.25)\right) + 0.25 \cdot \left(u(t-0.25) - u\left(t-\infty\right)\right)\right] \cdot V$$



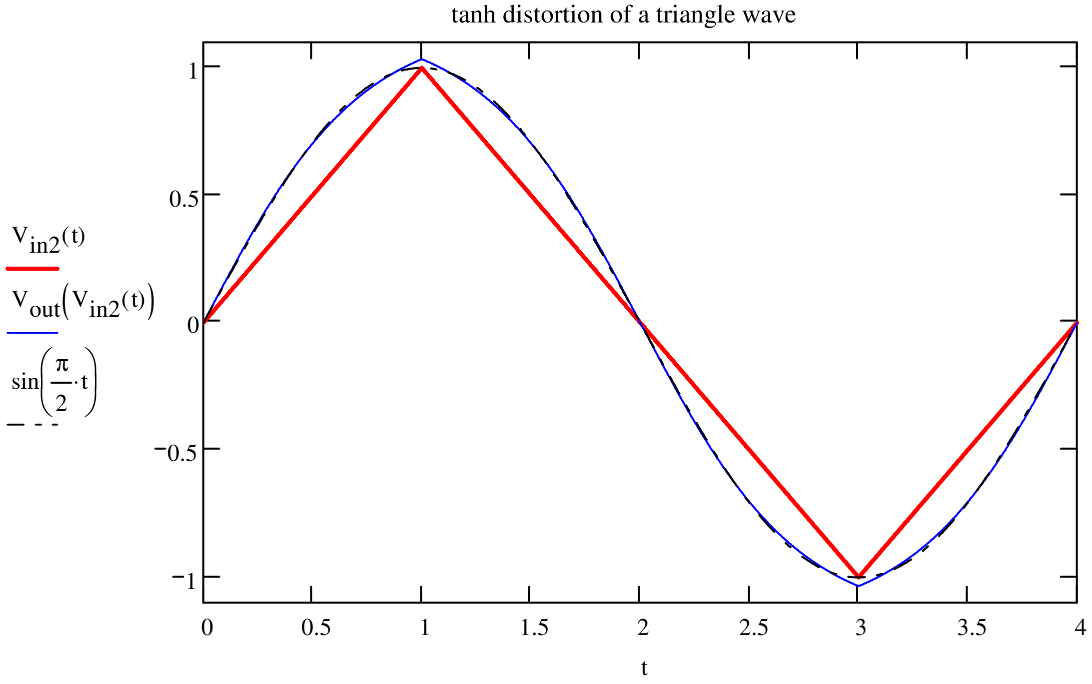
It should be noted that the output current is almost linear when the input voltage is very small. Even a 20mV signal is large for an OTA. In VCAs, this distortion can be a problem, but for the triangle to sine converter, this is exactly what we want. Next I'll show what happens with a triangle input. The following description of Vin builds the triangle wave out of 'unit step' functions shifted in time.

$$V_{in2}(t) := [t \cdot (u(t) - u(t-1)) + (2-t) \cdot (u(t-1) - u(t-3)) + (-4+t) \cdot (u(t-3) - u(t-4))] \cdot V$$

The distorted (sine) output is defined as the OTA output current with an attenuated version of the triangle wave as it's input. The output current from the OTA is sent through a load resistor to create a ~2Vp-p sine wave. I did some trial and error to come up with these values. Obviously the attenuation factor, and load resistor values are not realistic, but I chose them to get minimum distortion.

$V_{out}(V_{in}) := I_{out}\left(\frac{V_{in}}{13.855}\right)\cdot 2.344Kohm$ $V_{outb}(V_{in}) := I_{out}\left(\frac{V_{in}}{11.5}\right)\cdot 2127ohm$

The plot below demonstrates the tri->sine converter. The real sine wave is the dashed line. The only place it is much different from a real sine is at the sharp edges on the triangle wave. This plot looks comparable to the tri->sin converter I have built. I think that with some careful trimming, one could get very close to this result.



Now, I'll calculate the THD in the converted sine. The next calculations are used to find the amplitude spectrum of the converted sine.

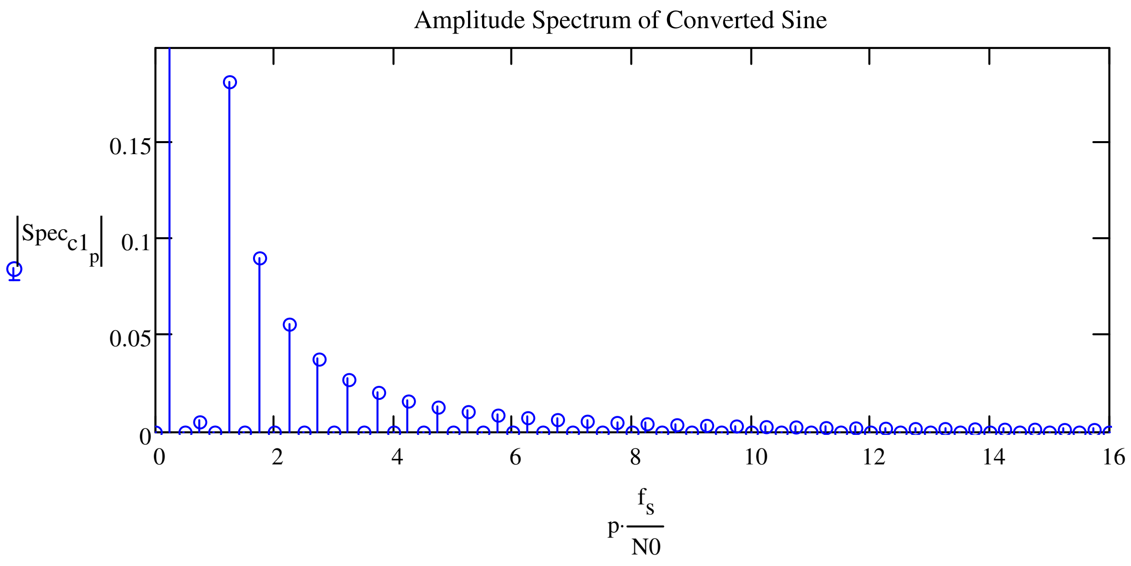
$N0 := 2^{10}$ $T0 := 4$ $f_s := \frac{N0}{4}$

$k := 0 .. N0 - 1$ $n_k := \frac{k}{f_s}$

$v_k := V_{out}(V_{in2}(n_k))\cdot\frac{1}{V}$ $Spec_{c1} := fft(v)$

$p := 0 .. \frac{N0}{2}$ $\max\left(\overrightarrow{\left|Spec_{c1}\right|}\right) = 15.999$

The amplitude spectrum plot is shown below. The first harmonic is the fundamental, and its amplitude is not shown. Instead, I have plotted over a range that shows the other harmonics more clearly. From this plot, it can be seen that the 3rd harmonic is not present in our converted sine.



$$v_{sin_k} := \sin\left(\frac{\pi}{2}\cdot n_k\right)$$

$$Spec_{sin} := \mathrm{fft}\left(v_{sin}\right)$$

$$p := 0 .. \frac{N0}{2}$$

$$\max\left(\overrightarrow{\left|Spec_{sin}\right|}\right) = 16$$

THD is calculated as the sum of powers in each harmonic of the output sine divided by the power in the fundamental frequency of a real sine. I have distortion expressed as a percentage. I was not able to achieve better results using the tanh distortion. I just guessed at values to find the best input level for the OTA. Some calculation could be done, but It wasn't obvious to me how to do that. The load resistor was chosen so that the first (fundametal) harmonic cancels out when a real sine wave is subtracted.

$$\frac{\mathrm{Im}\left(Spec_{c1_1}\right)}{\mathrm{Im}\left(Spec_{sin_1}\right)} = 1$$

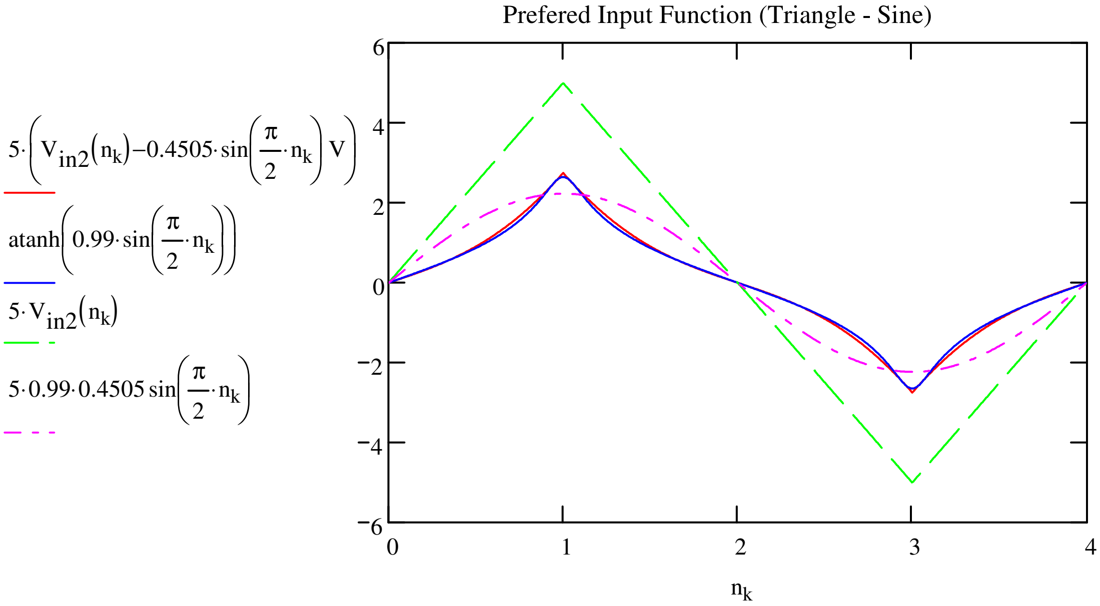
This shows that the fundamentals are equal and they will not affect our THD calculation

$$\mathrm{THD}(S) := \frac{\displaystyle\sum_{n=1}^{\frac{N0}{2}} \left|S_n\right|}{\left|Spec_{sin_1}\right|} - 1$$

$$\mathrm{THD}\left(Spec_{c1}\right) = 3.522\,\%$$

You may notice that the sharp corners were still in our sine wave. They can be removed by slightly increasing the signal level, but this will also increase the THD. If a smoother wave form is required, it could be easily achieved by adjusting the trim pots (you should have trim pots).

There is an easy way to improve the THD and remove the sharp corners at the same time. This is done using negative feedback. Someone mentioned it on the sdiy list reccently, but it was not until I wrote this note that I realized why it works. First, consider the best possible input to create a sine wave using tanh distortion. It will be arctanh(sine). A good estimate of arctanh(sine) can be achieved by subtracting a sine from a triangle. The next plot shows arctanh(sin) and triangle minus sine.



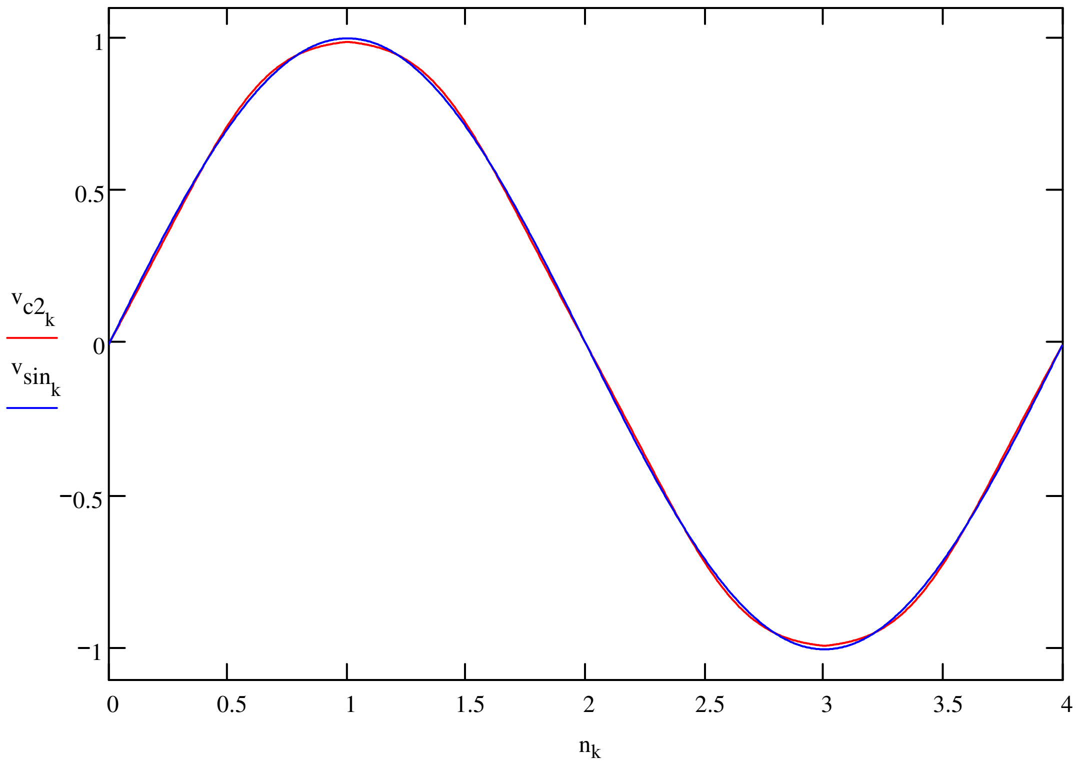
The equation that describes this new converter is:

$$V_{out}(t) = I_{abc}\cdot\tanh\left[\frac{\left(V_{in2}(t) - 0.4505\cdot V_{out}(t)\right)\cdot 0.26}{2\cdot V_t}\right]\cdot 1.994\cdot\mathrm{Kohm}$$

That is just like the first converter, but with negative feedback, and some values changed. The 0.4505 and 0.26 constants were chosen using the plot above so that the triangle minus sine would look just like the arctanh(sine). They were then fine tuned to minimize THD (trial and error again). The new load (1.994Kohm) was chosen so that the amplitude of the fundamental in the converted sine and the actual sine were equal.

In order to model this new converter, I could either solve the nonlinear equation numerically, or assume that it has a perfect sine wave at the output. If I can assume that it outputs a perfect sine wave then the Vout(t) in the right hand side can be replaced by sine(t*pi/2). Later, I will show a spice simulation to verify that this approximation works out. The new equation with a sine in the right had side is:

$$v_{c2_k} := \frac{I_{abc}}{A}\cdot\tanh\left[\frac{\left(V_{in2}(n_k) - 0.4505\cdot\sin\left(\frac{\pi}{2}\cdot n_k\right)\cdot V\right)\cdot .26}{2\cdot V_t}\right]\cdot 1.994\cdot\frac{\mathrm{Kohm}}{\mathrm{ohm}}$$

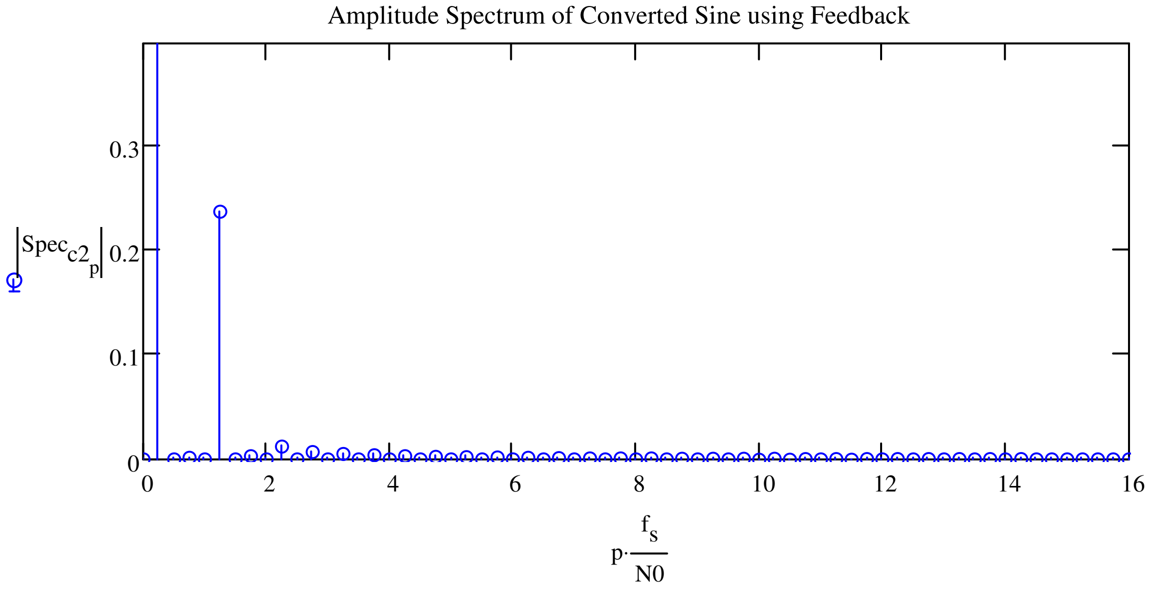


Actual and Converted Sine with Feedback

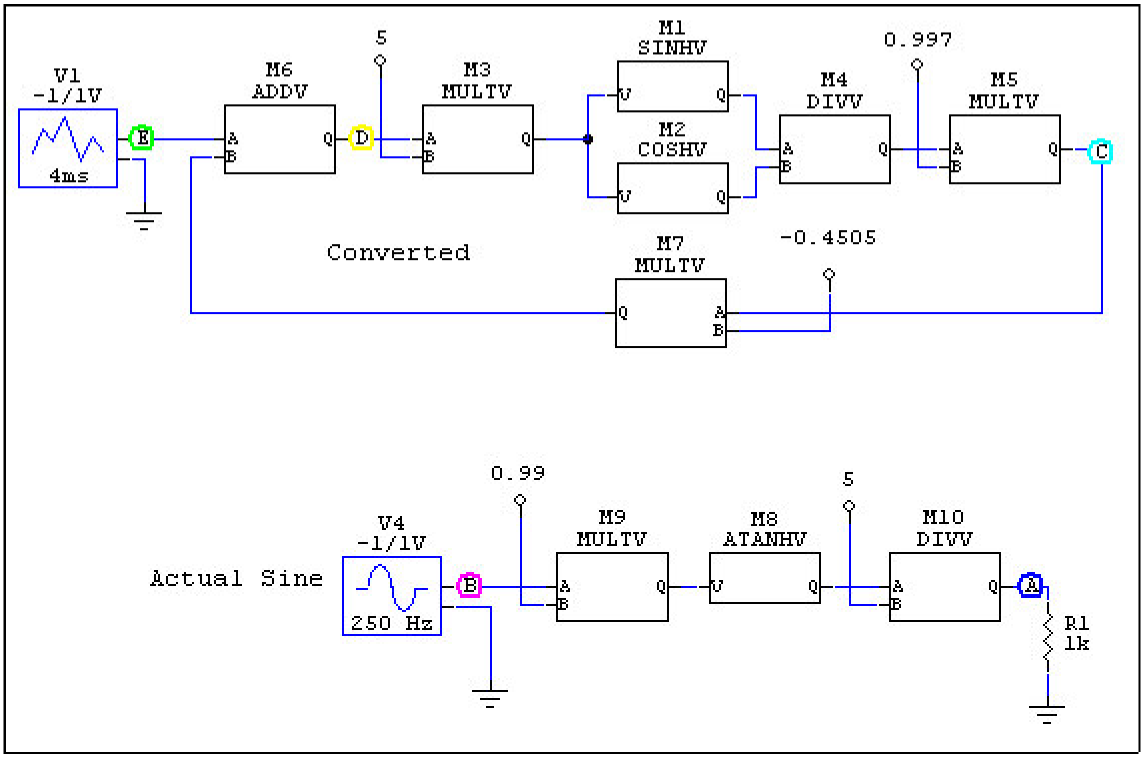
That is an improvement. It looks much better to the eye, but the THD is only about twice as good. In the spectrum plot below, only the 5th harmonic is significant. This is actually larger than in the first converter, but the high frequencies have been removed alone with the sharp corner.

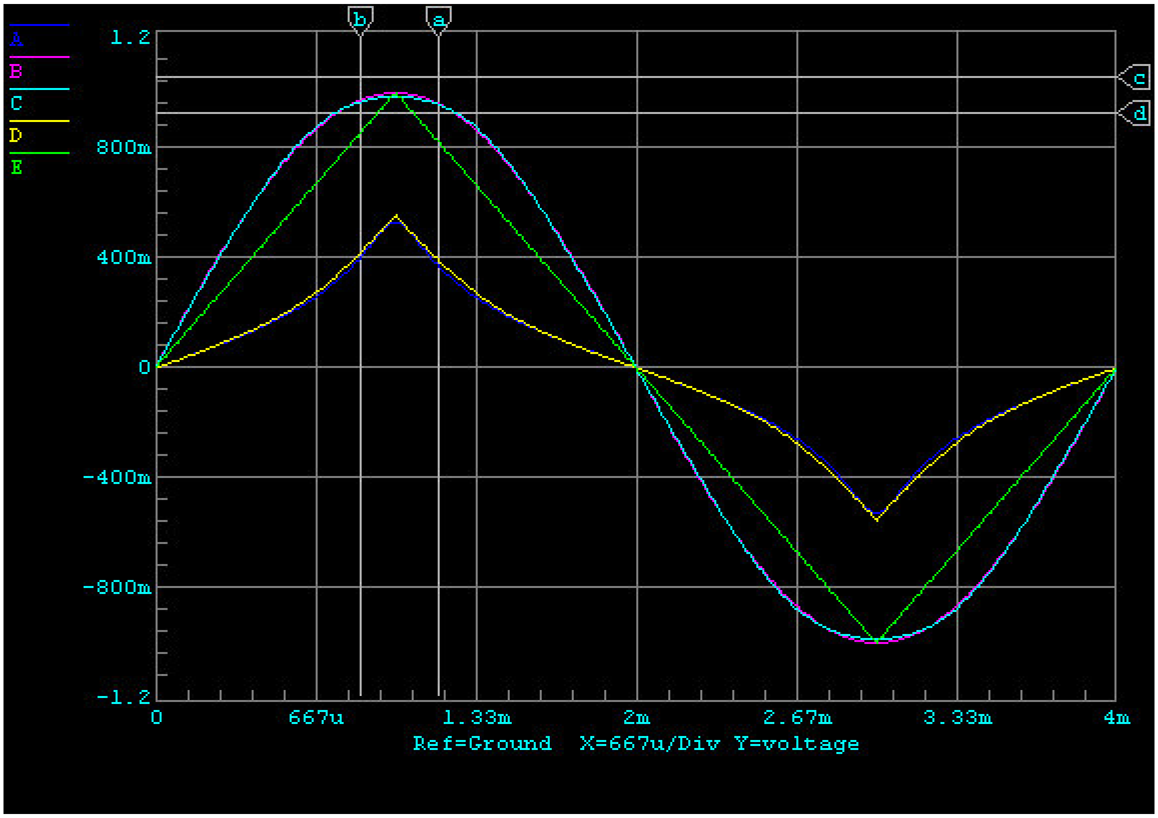
$Spec_{c2} := fft(v_{c2})$

$THD(Spec_{c2}) = 1.863\,\%$



To verify the operation. A spice simulation has been run. The OTA is represented using sinh/cosh because CircuitMaker student version does not have OTAs or tanh functions. It does have arctanh and that is compared against triangle minus sine.




A
B
C
D
E
b
a
c
d
1.2
800m
400m
0
-400m
-800m
-1.2
0
667u
1.33m
2m
2.67m
3.33m
4m
Ref=Ground X=667u/Div Y=voltage